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On the absence of BPS preonic solutions in IIA and IIB supergravities

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Abstract: We consider the present absence of $\nu = 31/32$ supersymmetric solutions in supergravity i.e., of solutions describing BPS preons. A recent result indicates that (bosonic) BPS preonic solutions do not exist in type IIB supergravity. We reconsider this analysis by using the G-frame method, extend it to the IIA supergravity case, and show that there are no (bosonic) preonic solutions for type IIA either. For the classical $D = 11$ supergravity no conclusion can be drawn yet, although the negative IIA results permit establishing the conditions that preonic solutions would have to satisfy. For supergravities with 'stringy' $(\alpha')^3$-corrections, the existence of BPS preonic solutions remains fully open.

Keywords: M-Theory, p-branes.

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1. Introduction

It has been argued in a very recent paper [1] that purely bosonic solutions preserving 31 out of 32 supersymmetries, hence describing *BPS preon* states [2], do not exist for IIB supergravity. Using the moving G-frame method of [3] (section 1.2), we rederive this result here (section 2). Then, we apply the same technique to the IIA case and also show that preonic solutions do not exist in type IIA supergravity (section 3). Nevertheless, the concluded absence of preonic solutions could be modified if the 'stringy' $(\alpha')^3$-corrections to the dilatino transformation rule were made explicit and taken into account (section 5).

For $D = 11$ supergravity, the existence of BPS preonic solutions is not ruled out even at the classical level (i.e., ignoring $(\alpha')^3$-corrections), although the above negative results for type IIA supergravity already set strong restrictions (section 4) to be satisfied by these solutions.

1.1 Basic notions and notation

In eleven-dimensional supergravity [4] the only fermionic field is the gravitino, $\check{\psi}^{\check{\alpha}} = dx^{\check{\mu}}\check{\psi}_{\check{\mu}}^{\check{\alpha}} = dx^{\mu}\check{\psi}_{\mu}^{\check{\alpha}} + dx^{\#}\check{\psi}_{\#}^{\check{\alpha}}$ ($\check{\mu} = (\mu; \#)$, $\mu = 0, 1, \ldots, 9$). In contrast, the ten-dimensional type II supergravities [5, 6] contain, in addition to two sixteen-component 'spin 3/2' gravitini, two 'spin 1/2' dilatini fields $\check{\chi}_{\check{\alpha}}$. We use the czek superscript $\check{\alpha}$ to denote the type II indices of the 32-component reducible spinors. In the IIB case $\check{\alpha}$ is the double index $\check{\alpha} = (\alpha, I)$, where $I = 1, 2$ labels the two Majorana-Weyl (MW) spinors of the same chirality and $\alpha = 1, \ldots 16$. In the IIA case, where both chiralities are present, $\check{\alpha}$ denotes the Majorana spinor index and thus $\check{\alpha} = 1, \ldots, 32$.

In particular, for the dilatino of type IIA supergravity we write

$$\text{IIA} : \qquad \check{\chi}^{\check{\alpha}} := (\chi^{\alpha 1}\ ,\ \chi^{2}_{\alpha})\ , \qquad \check{\alpha} = 1, \ldots 32\ , \quad \alpha = 1, \ldots 16\ . \tag{1.1}$$

while in type IIB supergravity the 32-component dilatino field decomposes into two MW spinors of the same chirality,

$$\text{IIB}\ :\qquad \check{\chi}_{\check{\alpha}} := (\chi^1_\alpha\ ,\ \chi^2_\alpha)\ ,\quad \check{\alpha} = (\alpha\,, I)\ ,\quad I = 1,2\ ,\quad \alpha = 1,\dots 16\ . \tag{1.2}$$

Notice that in the IIB case the position of the index $\check{\alpha}$ cannot be changed since the two MW spinors are of the same chirality and there is no 16×16 charge conjugation matrix in the MW spinor representation. In contrast, in type IIA a 32×32 charge conjugation matrix exists; it is anti-diagonal in the Weyl-like realization used here and exchanges the 1 and 2 MW components in (1.1).

In this condensed 32-component notation, the supersymmetry transformation rules for the gravitini and dilatini fermionic fields can be written in compact form for both IIA and IIB cases as

$$\delta_{\rm susy}\check{\psi}_a^{\check{\alpha}} = \mathcal{D}_a \check{\varepsilon}^{\check{\alpha}} := D_a\check{\varepsilon}^{\check{\alpha}} - \check{\varepsilon}^{\check{\beta}}\check{t}_{a\,\check{\beta}}{}^{\check{\alpha}}\ ,\qquad \delta_{\rm susy}\check{\chi} = \check{\varepsilon} M\ , \tag{1.3}$$

where $D = d - \omega$ is the Lorentz covariant derivative and $\mathcal{D} = D - \check{t}$ is the generalized covariant derivative which includes, besides the (suitable) spin connection $\omega := \frac{1}{4}\omega^{ab}\check{\Gamma}_{ab}$, the additional tensorial IIA or IIB $\check{t}$ contributions. The transformation rules for the dilatino are algebraic and are characterized by a 32×32 matrix $M_{\check{\beta}}{}^{\check{\alpha}}$. The form that this matrix takes will be crucial for the discussion below.

In the IIA case, and ignoring inessential bilinear fermionic contributions, the terms in $\delta_{\rm susy}\check{\chi}$ (eq. (1.3), see e.g. [6] and [7] and refs therein) are determined by the matrix

$$\text{IIA}\ :\qquad M_{\check{\beta}}{}^{\check{\alpha}} = \begin{pmatrix} \frac{3}{8}e^{\Phi}\not{R}^{(2)} + \frac{1}{8}\not{R}^{(4)} & \frac{1}{2}\not{\partial}\Phi - \frac{1}{4}\not{H}^{(3)} \\ \frac{1}{2}\tilde{\not{\partial}}\Phi + \frac{1}{4}\tilde{\not{H}}^{(3)} & -\frac{3}{8}e^{\Phi}\tilde{\not{R}}^{(2)} + \frac{1}{8}\tilde{\not{R}}^{(4)} \end{pmatrix}\ . \tag{1.4}$$

in terms of all the possible IIA fluxes (on-shell field strengths), namely,[1]

$$R_2 := dC_1\ ,\qquad R_4 := dC_3 - C_1\wedge H_3\ ,\qquad H_3 := dB_2 \qquad \textit{and} \qquad d\Phi\ : \tag{1.5}$$

$$\begin{cases} \not{H}^{(3)} = \frac{1}{3!}H_{abc}\sigma^{abc}\ , & \sigma^{abc} := (\sigma^{[a}\tilde{\sigma}^b\sigma^{c]})_{\alpha\beta}\ , \\ \tilde{\not{H}}^{(3)} = \frac{1}{3!}H_{abc}\tilde{\sigma}^{abc}\ , & \tilde{\sigma}^{abc} := (\tilde{\sigma}^{[a}\sigma^b\tilde{\sigma}^{c]})^{\alpha\beta} \end{cases}\ ,\qquad \begin{cases} \not{\partial}\Phi := \partial_a\Phi\sigma^a_{\alpha\beta}\ , \\ \tilde{\not{\partial}}\Phi := \partial_a\Phi\tilde{\sigma}^{a\,\alpha\beta}\ , \end{cases} \tag{1.6}$$

$$\begin{cases} \not{R}^{(2)} := \frac{1}{2!}R_{ab}(\sigma^{ab}) = -\tilde{\not{R}}^{(2)T}\ , & \sigma^{ab} := (\sigma^{[a}\tilde{\sigma}^{b]})_\alpha{}^\beta\ , \qquad \tilde{\sigma}^{ab} := (\tilde{\sigma}^{[a}\sigma^{b]})^\beta{}_\alpha \\ \not{R}^{(4)} = \frac{1}{4!}R_{abcd}\sigma^{abcd}{}_\alpha{}^\beta = (\tilde{\not{R}}^{(4)})^\beta{}_\alpha\ , & \sigma^{abcd} := (\sigma^{[a}\tilde{\sigma}^b\sigma^c\tilde{\sigma}^{d]})_\alpha{}^\beta\ . \end{cases} \tag{1.7}$$

The type IIB matrix M, in contrast, is given by (see [5])

$$\text{IIB}\ :\qquad M_{\check{\beta}\check{\alpha}} = \begin{pmatrix} \frac{1}{2}\not{\partial}\Phi + \frac{1}{4}\not{H}^{(3)} & -\frac{1}{2}e^{\Phi}\not{R}^{(1)} + \frac{1}{4}e^{\frac{1}{2}\Phi}\not{R}^{(3)} \\ \frac{1}{2}e^{\Phi}\not{R}^{(1)} + \frac{1}{4}e^{\frac{1}{2}\Phi}\not{R}^{(3)} & \frac{1}{2}\not{\partial}\Phi - \frac{1}{4}\not{H}^{(3)} \end{pmatrix}\ , \tag{1.8}$$

[1] $\sigma^a = \sigma^a_{\alpha\beta}$, $\tilde{\sigma}^a = \sigma^{a\,\alpha\beta}$, $a = 0,1,\dots,9$; $\sigma^a\tilde{\sigma}^b + \sigma^b\tilde{\sigma}^a = 2\eta^{ab} = \tilde{\sigma}^a\sigma^b + \tilde{\sigma}^b\sigma^a$. The sigma matrices with one and five (three) vector indices are symmetric (antisymmetric) with respect to the spinor ones. The transposition of untilded sigma matrices with four and two vector indices, respectively, converts them into the corresponding tilded and minus tilded ones.

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and involves the one-form and the three-form fluxes of type IIB supergravity,

$$R_1 := dC_0 \ , \quad R_3 := dC_2 - C_0 H_3 \ , \qquad H_3 := dB_2 \qquad and \qquad d\Phi \ , \tag{1.9}$$

but not the self dual five-form flux R_5,

$$R_5 := dC_4 - C_2 \wedge H_3 \ , \qquad R_5 = *R_5 \ \Leftrightarrow \begin{cases} \not{R}^{(5)} = 0 \ , \\ \tilde{\not{R}}^{(5)} \neq 0 \ . \end{cases} \tag{1.10}$$

When only purely bosonic solutions are considered, $\check{\psi} = 0$, $\check{\chi} = 0$, the parameter associated with the preserved supersymmetry obeys a differential equation and an algebraic one, namely $\mathcal{D}\check{\varepsilon} = 0$ and $\check{\varepsilon} M = 0$. Usually, to describe a solution preserving k supersymmetries (a $\nu = k/32$ state), one uses k independent bosonic Killing spinors $\epsilon_I^{\check{\alpha}}$ ($I = 1, \ldots k$, $\check{\varepsilon} = \kappa^I \epsilon_I^{\check{\alpha}}$ with arbitrary constant fermionic κ^I) that satisfy the following differential (from $\delta_{\mathrm{susy}} \check{\psi}_a^{\check{\alpha}} = 0$) and algebraic (from $\delta_{\mathrm{susy}} \check{\chi} = 0$) Killing equations

$$\mathcal{D}\check{\epsilon}_I := D\check{\epsilon}_I - \check{\epsilon}_I \check{t} \ = 0 \ , \tag{1.11}$$

$$\check{\epsilon}_I \, M \ = 0 \qquad (I = 1, \ldots, k) \ , \tag{1.12}$$

which guarantee that the solution remains bosonic and hence invariant after a gravitino and dilatino supersymmetry transformation.

The conclusion of [1] on the absence of a preonic solution of type IIB supergravity is based on the algebraic equation (1.12) and uses (1.11) to close the argument. We now recover this result below by using the moving G-frame method of [3].

1.2 The moving G-frame method and preonic spinors

A preonic state [2] preserves all supersymmetries but one; it is a $\nu = 31/32$ supersymmetric BPS state. As a result, it can be characterized by one bosonic spinor $\check{\lambda}_{\check{\alpha}}$ orthogonal to all the 31 bosonic Killing spinors $\check{\epsilon}_I^{\check{\alpha}}$ in (1.11),

$$\check{\epsilon}_I \, \check{\lambda} = \ \check{\epsilon}_I^{\check{\alpha}} \check{\lambda}_{\check{\alpha}} = 0 \ , \qquad I = 1, \ldots, 31 \ . \tag{1.13}$$

As it was noticed in [3], when the generalized holonomy group of supergravity [8, 9] is a subgroup of SL$(32, \mathbb{R})$ (which is the case for both D=11 [10] and type II D=10 supergravities [11]), the spinor characterizing a BPS preonic state obeys the differential equation

$$\mathcal{D}\check{\lambda} := D\check{\lambda} + \check{t}\check{\lambda} = 0 \ , \tag{1.14}$$

where $\check{t}$ is the same tensorial part of the generalized connection in eqs. (1.11) and (1.3). Notice that if $\check{t} \neq 0$ (the case of non-vanishing fluxes), eq. (1.14) is not equivalent to the Killing equation (1.11) even for the type IIA case where the 32×32 charge conjugation matrix does exist.

Applied to the present problem, the moving G-frame method [3] implies that eq. (1.12), looked at as an equation for the matrix M, is solved when $k = 31$ by

$$M = \check{\lambda} \otimes \check{s} \qquad i.e. \qquad \begin{cases} IIA \ : \ M_{\check{\beta}}{}^{\check{\alpha}} = \check{\lambda}_{\check{\beta}} \, \check{s}^{\check{\alpha}} \ , \\ IIB \ : \ M_{\check{\beta}\check{\alpha}} = \check{\lambda}_{\check{\beta}} \, \check{s}_{\check{\alpha}} \ , \end{cases} \tag{1.15}$$

where $\check{s}_{\breve{\alpha}}$ is a certain spinor. The algebraic structure of the matrix M implies a series of restrictions on the preonic spinor $\check{\lambda}_{\breve{\beta}}$. At the same time, eq. (1.15) imposes a series of restrictions on the fluxes involved in the matrix M.

Eq. (1.15) will be the basic equation in our analysis of the absence of preons among the bosonic solutions of type II supergravities.

2. Absence of preons in type IIB supergravity

In the type IIB case the matrix M has the form of eq. (1.8), and eq. (1.15) implies the following relations for the one- and three-form fluxes

$$\begin{aligned} \tfrac{1}{2}\not{\partial}\Phi + \tfrac{1}{4}\not{H}^{(3)} &= \lambda^1_\alpha s^1_\beta \,, && (2.1a)\\ -\tfrac{1}{2}e^{\Phi}\not{R}^{(1)} + \tfrac{1}{4}e^{\frac{1}{2}\Phi}\not{R}^{(3)} &= \lambda^1_\alpha s^2_\beta \,, && (2.1b)\\ +\tfrac{1}{2}e^{\Phi}\not{R}^{(1)} + \tfrac{1}{4}e^{\frac{1}{2}\Phi}\not{R}^{(3)} &= \lambda^2_\alpha s^1_\beta \,, && (2.1c)\\ \tfrac{1}{2}\not{\partial}\Phi - \tfrac{1}{4}\not{H}^{(3)} &= \lambda^2_\alpha s^2_\beta \,. && (2.1d) \end{aligned} \tag{2.1}$$

These fluxes then can be expressed through the IIB preonic spinor $\check{\lambda}_{\breve{\alpha}} := (\lambda^1_\alpha \ , \ \lambda^2_\alpha)$ and an arbitrary spinor $\check{s}_{\breve{\beta}} := (s^1_\beta \ , \ s^2_\beta)$. Furthermore, the consistency of eqs. (2.1) imposes a set of algebraic equations on these two spinors. They follow from the fact that the fluxes enter into eqs. (2.1) through matrices which possess definite symmetry properties,

$$(\not{\partial}\Phi)^T = +\not{\partial}\Phi \,, \qquad (\not{H}^{(3)})^T = -\not{H}^{(3)}, \qquad (\not{R}^{(3)})^T = -\not{R}^{(3)}, \qquad (\not{R}^{(1)})^T = +\not{R}^{(1)}. \tag{2.2}$$

These lead to the algebraic constraints

$$\begin{array}{llll} (a) & \lambda^1_{[\alpha}s^1_{\beta]} + \lambda^2_{[\alpha}s^2_{\beta]} = 0 \,, & (b) & -\lambda^1_{[\alpha}s^2_{\beta]} + \lambda^2_{[\alpha}s^1_{\beta]} = 0 \,, \\ (c) & \lambda^1_{(\alpha}s^1_{\beta)} - \lambda^2_{(\alpha}s^2_{\beta)} = 0 \,, & (d) & \lambda^1_{(\alpha}s^2_{\beta)} + \lambda^2_{(\alpha}s^1_{\beta)} = 0 \,. \end{array} \tag{2.3}$$

A straightforward algebra shows that eqs. (2.3) have only trivial solutions. This means that either the preonic or the auxiliary spinor is zero,

$$\text{IIB} \ : \qquad \lambda^1_\alpha = \lambda^2_\alpha = 0 \qquad or \qquad s^1_\beta = s^2_\beta = 0 \,. \tag{2.4}$$

In both cases the matrix $M = 0$ and, hence, all the fluxes except the five-form flux (eq. (1.10)) are equal to zero, $R_1 = d\Phi = R_1 = R_3 = 0$. Nevertheless, the fact that the solution $s^1_\beta = s^2_\beta = 0$ of (2.3) allows for a non-vanishing preonic spinor $(\lambda^1_\alpha, \lambda^2_\alpha)$ might give hope, at this stage, of finding a nontrivial and unique solution $\check{\lambda}$ to eq. (1.14) and $k = 31$ solutions $\check{\epsilon}_I$ for eq. (1.11). This possibility is ruled out by looking at eq. (1.11). For simplicity let us begin by discussing eq. (1.14). When only the five-form flux is non-vanishing, eq. (1.14) would acquire the relatively simple form of

$$R_1 = R_3 = H_3 = d\Phi = 0 \ : \qquad \left\{ \begin{array}{l} D_b\lambda^1_\alpha = -\frac{1}{16}(\sigma_b \not{R}^{(5)})_\alpha{}^\beta \lambda^2_\beta \,, \\ D_b\lambda^2_\alpha = \ \frac{1}{16}(\sigma_b \not{R}^{(5)})_\alpha{}^\beta \lambda^1_\beta \,. \end{array} \right. \tag{2.5}$$

Now one observes that, if $(\lambda^1_\alpha, \lambda^2_\alpha)$ is a solution of eq. (2.5), $(-\lambda^2_\alpha, \lambda^1_\alpha)$ provides another one. As a result, the number of solutions of eqs. (2.5) is always even. The same is true of the Killing equation (1.11) since it has the same structure. Hence with vanishing one- and three-form fluxes one can only have an even number of preserved supersymmetries. These might include two-preonic solutions (preserving 30 supersymmetries) besides those preserving all 32 supersymmetries, but not a preonic solution. The authors of [1] then concluded that preonic solutions do not exist for type IIB supergravity.

We now apply our G-frame approach, used above to rederive the IIB result of [1], to show that preonic solutions are also absent in type IIA supergravity.

3. Absence of preons in type IIA supergravity

The crucial point is that in the IIA case the matrix M, eq. (1.4), receives contributions from all IIA fluxes, eq. (1.5). Hence if M is zero, all IIA fluxes are zero, the generalized covariant derivative $\mathcal{D}$ becomes the Lorentz covariant derivative D and the generalized holonomy group reduces to $\mathrm{SO}(1,9)$, for which the number of possible preserved supersymmetries is known (see [9, 12]).

As we shall see presently, M is indeed zero if we assume the existence of 31 Killing spinors. In type IIA supergravity the preonic $\check{\lambda}_{\check{\alpha}}$ and auxiliary $\check{s}^{\check{\alpha}}$ spinors are 32-component $D=10$ Majorana spinors,

$$\text{IIA}: \qquad \check{\lambda}_{\check{\alpha}} := (\lambda^1_\alpha\ ,\ \lambda^{\alpha 2})\ , \qquad \check{s}^{\check{\alpha}} := (s^{\alpha 1}\ ,\ s^2_\alpha)\ , \qquad \alpha = 1, \ldots, 16\ . \tag{3.1}$$

Eq. (1.15) can be split into four equations for the (16×16)-component blocks

$$\begin{aligned} &\tfrac{3}{8}e^{\Phi}\not{R}^{(2)} + \tfrac{1}{8}\not{R}^{(4)} = \lambda^1_\alpha s^{\beta 1}\ , \quad (3.2\text{a}) \qquad &&\tfrac{1}{2}\not\partial\Phi - \tfrac{1}{4}\not{H}^{(3)} = \lambda^1_\alpha s^2_\beta\ , \quad (3.2\text{b}) \\ &\tfrac{1}{2}\tilde{\not\partial}\Phi + \tfrac{1}{4}\tilde{\not{H}}^{(3)} = \lambda^{\alpha 2}s^{\beta 1}\ , \quad (3.2\text{c}) \qquad &&-\tfrac{3}{8}e^{\Phi}\tilde{\not{R}}^{(2)} + \tfrac{1}{8}\tilde{\not{R}}^{(4)} = \lambda^{\alpha 2}s^2_\beta\ . \quad (3.2\text{d}) \end{aligned} \tag{3.2}$$

We now notice that $\tilde{\not{R}}^{(2)} = -(\not{R}^{(2)})^T$, $\tilde{\not{R}}^{(4)} = +(\not{R}^{(4)})^T$ and that, accordingly, the *l.h.s.*'s of eqs. (3.2a) and (3.2d) are equal among themselves. Hence, the *r.h.s.*'s of these equations are also equal, $\lambda^1_\alpha s^{\beta 1} = \lambda^{\beta 2}s^1_\alpha$. This equation identifies the components of $\check{\lambda}$ and $\check{s}$ up to a factor a,

$$s^{\alpha 1} = a\lambda^{\alpha 2}\ , \qquad s^2_\alpha = a\lambda^1_\alpha\ . \tag{3.3}$$

Then, decomposing eq. (3.2a) or (3.2d) into their irreducible parts (i.e., identifying the coefficients of the matrices ${\sigma^{ab}}_\alpha{}^\beta$, ${\sigma^{abcd}}_\alpha{}^\beta$ *and* $\delta_\alpha{}^\beta$, one finds the expressions for the RR fluxes in terms of preonic spinors as well as an orthogonality condition between λ^1 and λ^2,

$$R_{ab} = -\frac{a}{6}e^{-\Phi}\lambda^2\sigma_{ab}\lambda^1\ , \qquad R_{abcd} = -\frac{a}{2}\lambda^2\sigma_{abcd}\lambda^1\ , \qquad \lambda^{\alpha 2}\lambda^1_\alpha\ = 0. \tag{3.4}$$

Substituting (3.3) for the s spinors in (3.2b) and (3.2c), these equations can be rewritten in the form

$$\frac{1}{2}\not\partial\Phi - \frac{1}{4}\not{H}^{(3)} = a\lambda^1_\alpha\lambda^1_\beta\ , \quad (3.5\text{a}) \qquad \frac{1}{2}\tilde{\not\partial}\Phi + \frac{1}{4}\tilde{\not{H}}^{(3)} = a\lambda^{\alpha 2}\lambda^{\beta 2}\ . \quad (3.5\text{b}) \tag{3.5}$$

The *r.h.s.*'s of eqs. (3.5) are symmetric, while the *l.h.s.*'s contain the antisymmetric matrices $\not{H}^{(3)} = -(\not{H}^{(3)})^T$ and $\tilde{\not{H}}^{(3)} = -(\tilde{\not{H}}^{(3)})^T$ which, hence, should be equal to zero. This

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implies the vanishing of the NS-NS flux H_3 for a hypothetical preonic solution of type IIA supergravity, $H_{abc} = 0$. Then one arrives at

$$\frac{1}{2}\sigma^a_{\alpha\beta} D_a \Phi = a\lambda^1_\alpha \lambda^1_\beta \,, \qquad \frac{1}{2}\tilde{\sigma}^{a\alpha\beta} D_a \Phi = a\lambda^{\alpha 2}\lambda^{\beta 2} \,. \tag{3.6}$$

Since we are in ten dimensions these equations imply, besides $D_a\Phi \sim \lambda^1 \tilde{\sigma}_a \lambda^1$,

$$a\lambda^1 \tilde{\sigma}^{a_1\ldots a_5}\lambda^1 = 0 \,, \qquad a\lambda^2 \sigma^{a_1\ldots a_5}\lambda^2 = 0 \,. \tag{3.7}$$

Eqs. (3.6) or (3.7) imply the *absence of BPS preons among the bosonic solutions of type IIA supergravity.* Indeed, for non-vanishing a ($a \neq 0$) eqs. (3.7) have only trivial[2] solutions, $\lambda^1 = 0 = \lambda^2$. This may correspond to the case of a fully supersymmetric solution of supergravity (preserving the 32 supersymmetries), but not to a preonic one. The other possibility, $a = 0$, also implies the vanishing of the M matrix (1.4) and hence of all type IIA supergravity fluxes, $R_2 = 0 = R_4$, $H_3 = 0 = d\Phi$, and thus the generalized connection in the Killing equation (1.11) reduces to the spin-connection, $\mathcal{D} = D$. In such a case it is known (see [9, 12]) that the Killing spinor equation $D\check{\epsilon} = 0$ may have either 32 or up to 16 solutions. Thus a solution preserving 31 supersymmetries, a BPS preonic solution, is not allowed.

This completes the proof of the absence of BPS preonic, $\nu = 31/32$ supersymmetric bosonic solutions in type II supergravities i.e., in the *classical* approximation to the type II string theories.

4. The case of $D = 11$ supergravity

It is known that the $D = 10$ type IIA supergravity can be obtained by dimensional reduction from $D = 11$ supergravity i.e., its solutions can be identified with solutions of $D = 11$ supergravity that are independent of one of the coordinates. In particular, the type IIA dilatino $\check{\chi}^{\check{\alpha}}$, eq. (1.1), originates from the 11-th component $\check{\psi}^{\check{\alpha}}_{\#}$ of the $D = 11$ gravitino $\check{\psi}^{\check{\alpha}}_{\check{\mu}} = (\check{\psi}^{\check{\alpha}}_{\mu}, \check{\psi}^{\check{\alpha}}_{\#})$; schematically,

$$\check{\chi}^{\check{\alpha}} = \check{\psi}^{\check{\alpha}}_{\#} \,. \tag{4.1}$$

The type IIA supersymmetry transformations can also be obtained from those of $D = 11$ by dimensional reduction . This implies, in particular, that the IIA M-matrix (1.4) comes from the eleventh component of the $D = 11$ generalized connection; schematically,

$$M_{\check{\beta}}{}^{\check{\alpha}} = (\omega + \check{t})_{\#\,\check{\beta}}{}^{\check{\alpha}} \,. \tag{4.2}$$

[2] A simple way to prove it from eq. (3.6) is to notice that this equation implies $D_a\Phi \propto \lambda^1\tilde{\sigma}_a\lambda^1$ and that, hence, $D_a\Phi$ is a light-like ten-vector, $D_a\Phi D^a\Phi = 0$. Then one may choose the Lorentz frame where $D_a\Phi \propto (1, 0, \ldots, 0, \pm 1)$; in it, $D_a\Phi\sigma^a_{\alpha\beta} \propto (\sigma^0_{\alpha\beta} \pm \sigma^9_{\alpha\beta}) = 2\sum_p \delta^p_\alpha \delta^p_\beta$, where $p = 1, \ldots, 8$. In this frame, the first equation in (3.6) reads $D_0\Phi \sum_p \delta^p_\alpha\delta^p_\beta = a\lambda^1_\alpha\lambda^1_\beta$, which immediately implies that $a \neq 0$ is only possible if half of the sixteen components of λ^1_β are zero, $\lambda^1_\beta = \lambda_q \delta^q_\beta$. Taking this in account, the above equation reduces to $D_0\Phi\delta_{qp} = a\lambda_q\,\lambda_p$ with $p, q = 1, \ldots, 8$, which for $a \neq 0$ only admits the trivial solution $\lambda^1 = 0 = \lambda^2$.

This observation provides a starting point to probe the existence of BPS preonic solutions in $D = 11$ supergravity or, more precisely, among the purely bosonic solutions of the classical $D = 11$ supergravity [4]. It was shown in [13] that the existence of k Killing *spinors* ($k = 31$ for preonic solutions) implies the existence of $k(k+1)/2$ Killing *vectors*,

$$K^{\check{a}}_{IJ} := \check{\epsilon}_I^{\check{\alpha}} \Gamma^{\check{a}}_{\check{\alpha}\check{\beta}} \check{\epsilon}_J^{\check{\beta}} \ , \tag{4.3}$$

such that both the metric and the field strength $F_4 = dA_3$ of the three-form gauge field A_3 are invariant under 'translations' along the directions of $K^{\check{a}}_{IJ}$,

$$\delta_{K_{IJ}} g_{\check{\mu}\check{\nu}} = 2D_{(\check{\mu}} K_{\check{\nu})IJ} = 0 \ , \qquad \delta_{K_{IJ}} F_4 := \mathcal{L}_{K_{IJ}} F_4 = 0 \ . \tag{4.4}$$

This actually implies that any supersymmetric solution of $D = 11$ can be considered (at least locally) as a solution of $D = 10$ type IIA supergravity lifted ('oxidized') to $D = 11$. Thus, because of the above negative result for the existence of preonic solutions in type IIA supergravity, the only remaining possibility to have BPS preonic solutions in the D=11 case requires that they result from the 'oxidization' of a less supersymmetric solution of the $D = 10$ type IIA supergravity.

If the lifting to $D = 11$ has to produce more supersymmetries, we need that one or more Killing spinors $\epsilon_I^{\check{\alpha}}$ have non-vanishing derivative in the direction of a Killing vector, schematically, $\partial_\# \epsilon_I^{\check{\alpha}} \neq 0$. In this way, the set of $D = 11$ Killing equations $\mathcal{D}_\# \epsilon_I^{\check{\alpha}} := D_\# \epsilon_I^{\check{\alpha}} - \epsilon_I^{\check{\beta}} \check{t}_{\# \check{\beta}}{}^{\check{\alpha}} = 0$ will no longer reduce (see eq. (4.2)) to the algebraic equation (1.12). As a result, the arguments from the discussion of the type IIA case would not apply in $D = 11$ to exclude the existence of a preonic solution.

A Killing spinor $\epsilon_J^{\check{\alpha}}$ can be characterized [13] by means of three differential forms: a Killing vector one-form $K_{1\,JJ} := e_{\check{a}} K^{\check{a}}_{JJ}$, a two-form $\Omega_{2\,JJ}$ and a five-form $\Sigma_{5\,JJ}$. These forms are the diagonal elements of the symmetric bilinear matrix forms with tensorial components defined in eq. (4.3) and by

$$\Omega^{\check{a}_1\check{a}_2}_{IJ} := \check{\epsilon}_I^{\check{\alpha}} \Gamma^{\check{a}_1\check{a}_2}_{\check{\alpha}\check{\beta}} \check{\epsilon}_J^{\check{\beta}} \ , \qquad \Sigma^{\check{a}_1\ldots\check{a}_5}_{IJ} := \check{\epsilon}_I^{\check{\alpha}} \Gamma^{\check{a}_1\ldots\check{a}_5}_{\check{\alpha}\check{\beta}} \check{\epsilon}_J^{\check{\beta}} \ . \tag{4.5}$$

The independence of a Killing spinor on a coordinate $x^\#$ in some direction would also imply the independence of its associated Killing vector K_{JJ} (eq. (4.3)), of the two-form Ω_{JJ} and of the five-form Σ_{JJ} (eq. (4.4)) on that direction. As the direction $x^\#$ should be characterized by one of the Killing vectors, the result of [13], stating that $\mathcal{L}_K \Omega_2 = 0$ and $\mathcal{L}_K \Sigma_5 = 0$, implies the independence of the two- and the five-form on $x^\#$. However, the Lie derivative of a Killing vector with respect to another Killing vector, $\mathcal{L}_K K'_1$, may still be nonzero when there are two or more Killing vectors. Thus, at present we cannot conclude that all Killing spinors $\epsilon_I^{\check{\alpha}}$ are independent of $x^\#$ so that, albeit rather exotic, the possibility of a $\nu = 31/32$ supersymmetric solution in $D = 11$ supergravity remains open.

5. Could preonic BPS solutions still exist?

The established absence of preonic solutions in type II supergravities, i.e. for the *classical* approximations to type II string theories, does not preclude the preonic conjecture of [2].

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At the time it was made, solutions preserving more than 16 out of the 32 supersymmetries were not known except for the fully supersymmetric ones (see [14]). It was already mentioned in [2] that a kind of 'BPS preon conspiracy' could produce that only composites of some number of preons (but not the preons themselves) could be found ('observed') as supergravity solutions.

On account of the fundamental role played by preons in the classification of BPS states [2], it is tempting to speculate that the fact that type II supergravities do not have preonic solutions rather points out at a need for their modification. The most natural refinement to try is to take into account stringy, $(\alpha')^3$-corrections to the supergravity equations and to the supersymmetry transformation rules of the supergravity fields. Preonic solutions in a 'stringy corrected' type IIA supergravity would be allowed if the corrections modified eqs. (3.6) by adding some terms $\propto \sigma_{abcdf}$ and $\propto \tilde{\sigma}_{abcdf}$. Schematically, the 'required' modification would have to be of the form

$$\begin{aligned} a\lambda^1_\alpha\lambda^1_\beta - \frac{1}{2}\sigma^a_{\alpha\beta}D_a\Phi = 0 \quad &\mapsto \quad a\lambda^1_\alpha\lambda^1_\beta - \frac{1}{2}\sigma^a_{\alpha\beta}D_a\Phi = Q^-_{abcde}\sigma^{abcde}_{\alpha\beta}\,, \\ a\lambda^{2\alpha}\lambda^{2\beta} - \frac{1}{2}\tilde{\sigma}^{a\,\alpha\beta}D_a\Phi = 0 \quad &\mapsto \quad a\lambda^{2\alpha}\lambda^{2\beta} - \frac{1}{2}\tilde{\sigma}^{a\,\alpha\beta}D_a\Phi = Q^+_{abcde}\tilde{\sigma}^{abcde\,\alpha\beta} \end{aligned} \tag{5.1}$$

for some $Q_{abcde} \propto (\alpha')^3$ (clearly, the $\propto \sigma_a$ contribution could also be changed, but this is not essential for the present schematic discussion). Such a modification (5.1) of eq. (3.6) might result from the associated additions to the dilatino transformation rules (1.3) (of the type $\propto \tilde{\sigma}_{abcdf}$ plus other terms not essential for our discussion). In terms of the M matrix, this modification would imply

$$M \;\mapsto\; M + \begin{pmatrix} 0 & Q^-_{abcde}\sigma^{abcde}_{\alpha\beta} \\ Q^+_{abcde}\tilde{\sigma}^{abcde\,\alpha\beta} & 0 \end{pmatrix} = M + Q^\pm_{abcde}\Gamma^{abcde}\frac{1}{2}(1 \pm \Gamma^{11})\,. \tag{5.2}$$

Direct calculations of the stringy corrections [15 − 17] to the supersymmetry transformation rules have been hampered by the lack of a covariant technique to calculate higher order loop amplitudes in superstring theory[3]. Nevertheless, bosonic string calculations allowed to find stringy corrections to the Einstein equation [17]. The influence of these corrections on the supersymmetric vacua and their relevance for their supersymmetric properties [19] was used to find corrections to the gravitino supersymmetry transformation properties. As the discussions of the α' modifications have also been extended to the eleven-dimensional theory [16, 20][4], one can obtain the 'corrected' transformation rules for the type IIA dilatino[5] by dimensional reduction from those of the $D = 11$ gravitino and thus derive the expression of the matrix M in eq. (1.4) that incorporates the 'stringy corrected' counterpart of eq. (4.2).

[3]Such a technique has been recently proposed in the framework of Berkovits's pure spinor approach [18] to the covariant description of the quantum superstring.

[4]The contributions to the generalized connection (i.e. to the supersymmetry transformation rules for the gravitino) were calculated for a particular background and, then, conjectured to hold in general [20] on grounds of their universal form.

[5]For the heterotic string case, the simplest possible corrections to the $(N = 1)$ dilatino χ transformation rules (see eq. (23) in [19]) consist in modifying ('renormalizing') the dilaton Φ appearing in the standard supersymmetry transformations.

In this perspective it looks promising that the $D=11$ generalized connection $\check{t}_{\tilde{\mu}} = (\check{t}_{\mu}, \check{t}_{10})$ (*cf.* (1.3); here $\tilde{\mu} = (\mu; \#) = 0, \ldots 9; 10$) considered in [20], contains the terms $\check{Q}^{\tilde{\mu}_1 \ldots \tilde{\mu}_6} \Gamma_{\tilde{\mu}_1 \ldots \tilde{\mu}_6}$; their dimensional reduction would produce, among others, the contribution $\check{Q}^{\mu_1 \ldots \mu_5\, 10} \Gamma_{\mu_1 \ldots \mu_5} \Gamma^{11}$ which is of the needed type, see eq. (5.2) (the $\Gamma^{10} \equiv \Gamma^{\#}$ in $D=11$ is the Γ^{11} in $D=10$).

To summarize, although it has been shown that a $\nu = 31/32$ preonic solution is not allowed in the *classical* type II supergravities (in [1] for type IIB and here for type IIA), a conclusive analysis with quantum stringy corrections, providing a more precise description of string/M-theory, remains to be done. If preons were found to exist when quantum corrections are taken into account, it would be only natural on account of their special role as the 'quarks of M-theory' [2] [6]. Preons would only be 'seen' by looking at the 'quantum solutions' of string theory, an approximation of which is provided by supergravity with stringy corrections.

As far as the study of 'classical' supergravity is concerned, the natural next step is to clarify the level of the mentioned 'preon conspiracy' [2] in the classical $D=10$ supergravity *i.e.*, whether it is possible to find two-preonic $\nu = 30/32$ supersymmetric solutions, preserving all but two supersymmetries, or whether the 'counterpart' of the colourless quark states in the case of preons should include no less than four preonic constituents corresponding to the highest non-fully supersymmetric states up to now found, the $\nu = 28/32$ states of the IIB case [21].

As for $D=11$ supergravity, although we have not been able to reach a definite conclusion on the existence of $\nu = 31/32$ supersymmetric solutions, we have presented here their characteristic properties: such a $D=11$ BPS preonic solution should have Killing directions, both for the metric g and the gauge field strength F_4, such that at least one of its 31 Killing spinors depends on the coordinates corresponding to these directions.

Finally, we conclude by mentioning that all searches for preonic solutions, including this one, have been concerned with purely bosonic solutions, a restriction that does not follow from [2].

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References

[1] U. Gran, J. Gutowski, G. Papadopoulos and D. Roest, $N=31$ *is not IIB*, `hep-th/0606049`.

[6] Let us recall [2] that the potential relevance of BPS preons derives from their quark-like role in the classification of BPS states according to the number of preserved supersymmetries: all BPS states of M-theory preserving k supersymmetries can be considered as composites of $32-k$ BPS preons (the statement is true for any D with 32 replaced by the corresponding spinor dimension).

[2] I.A. Bandos, J.A. de Azcárraga, J.M. Izquierdo and J. Lukierski, *BPS states in M-theory and twistorial constituents*, *Phys. Rev. Lett.* **86** (2001) 4451 [hep-th/0101113].

[3] I.A. Bandos, J.A. de Azcárraga, J.M. Izquierdo, M. Picon and O. Varela, *On BPS preons, generalized holonomies and $d = 11$ supergravities*, *Phys. Rev.* **D 69** (2004) 105010 [hep-th/0312266].

[4] E. Cremmer, B. Julia and J. Scherk, *Supergravity theory in 11 dimensions*, *Phys. Lett.* **B 76** (1978) 409.

[5] J.H. Schwarz and P.C. West, *Symmetries and transformations of chiral $N = 2$ $D = 10$ supergravity*, *Phys. Lett.* **B 126** (1983) 301;
J.H. Schwarz, *Covariant field equations of chiral $N = 2$ $D = 10$ supergravity*, *Nucl. Phys.* **B 226** (1983) 269;
P.S. Howe and P.C. West, *The complete $N = 2$, $D = 10$ supergravity*, *Nucl. Phys.* **B 238** (1984) 181.

[6] F. Giani and M. Pernici, *$N = 2$ supergravity in ten-dimensions*, *Phys. Rev.* **D 30** (1984) 325;
I.C.G. Campbell and P.C. West, *$N = 2$ $D = 10$ nonchiral supergravity and its spontaneous compactification*, *Nucl. Phys.* **B 243** (1984) 112;
M. Huq and M.A. Namazie, *Kaluza-Klein supergravity in ten-dimensions*, *Class. and Quant. Grav.* **2** (1985) 293.

[7] D. Marolf, L. Martucci and P.J. Silva, *Actions and fermionic symmetries for D-branes in bosonic backgrounds*, *JHEP* **07** (2003) 019 [hep-th/0306066].

[8] M.J. Duff and K.S. Stelle, *Multi-membrane solutions of $D = 11$ supergravity*, *Phys. Lett.* **B 253** (1991) 113.

[9] M.J. Duff and J.T. Liu, *Hidden spacetime symmetries and generalized holonomy in M-theory*, *Nucl. Phys.* **B 674** (2003) 217 [hep-th/0303140].

[10] C. Hull, *Holonomy and symmetry in M-theory*, hep-th/0305039.

[11] G. Papadopoulos and D. Tsimpis, *The holonomy of IIB supercovariant connection*, *Class. and Quant. Grav.* **20** (2003) L253 [hep-th/0307127].

[12] R.L. Bryant, *Pseudo-Riemannian metrics with parallel spinor fields and vanishing Ricci tensor*, Sémin. Congr., 4, Soc. Math. France, Paris, 2000, 53-94 [math.DG/0004073].

[13] J.P. Gauntlett and S. Pakis, *The geometry of $D = 11$ Killing spinors*, *JHEP* **04** (2003) 039 [hep-th/0212008].

[14] J. Figueroa-O'Farrill and G. Papadopoulos, *Maximally supersymmetric solutions of ten- and eleven-dimensional supergravities*, *JHEP* **03** (2003) 048 [hep-th/0211089].

[15] M.T. Grisaru, A.E.M. van de Ven and D. Zanón, *Two-dimensional supersymmetric sigma models on Ricci flat Kähler manifolds are not finite*, *Nucl. Phys.* **B 277** (1986) 388;
C.N. Pope, M.F. Sohnius and K.S. Stelle, *Counterterm counterexamples*, *Nucl. Phys.* **B 283** (1987) 192;
A. Candiello, K. Lechner and M. Tonin, *k anomalies and space-time supersymmetry in the Green-Schwartz heterotic superstring*, *Nucl. Phys.* **B 438** (1995) 67 [hep-th/9409107].

[16] K. Peeters, P. Vanhove and A. Westerberg, *Supersymmetric higher-derivative actions in ten and eleven dimensions, the associated superalgebras and their formulation in superspace*, *Class. and Quant. Grav.* **18** (2001) 843 [hep-th/0010167], and refs. therein and in [20].

[17] D.J. Gross and E. Witten, *Superstring modifications of Einstein's equations*, *Nucl. Phys.* **B 277** (1986) 1.

[18] N. Berkovits, *Multiloop amplitudes and vanishing theorems using the pure spinor formalism for the superstring*, *JHEP* **09** (2004) 047 [hep-th/0406055]; *Super-Poincaré covariant two-loop superstring amplitudes*, *JHEP* **01** (2006) 005 [hep-th/0503197];
N. Berkovits and C.R. Mafra, *Equivalence of two-loop superstring amplitudes in the pure spinor and rns formalisms*, *Phys. Rev. Lett.* **96** (2006) 011602 [hep-th/0509234].

[19] P. Candelas, M.D. Freeman, C.N. Pope, M.F. Sohnius and K.S. Stelle, *Higher order corrections to supersymmetry and compactifications of the heterotic string*, *Phys. Lett.* **B 177** (1986) 341.

[20] H. Lu, C.N. Pope, K.S. Stelle and P.K. Townsend, *String and M-theory deformations of manifolds with special holonomy*, *JHEP* **07** (2005) 075 [hep-th/0410176];
H. Lu, C.N. Pope and K.S. Stelle, *Generalised holonomy for higher-order corrections to supersymmetric backgrounds in string and M-theory*, *Nucl. Phys.* **B 741** (2006) 17 [hep-th/0509057].

[21] I. Bena and R. Roiban, *Supergravity pp-wave solutions with 28 and 24 supercharges*, *Phys. Rev.* **D 67** (2003) 125014 [hep-th/0206195].